FALL 2019: MATH 558 QUIZ 11 SOLUTIONS

Each question is worth 5 points.

1. Define the concept of a group.

Solution. A group is a set G together with a binary operation $\cdot$ satisfying the following conditions:

(i) There exists $e \in G$ such that $g \cdot e = g = e \cdot g$, for all $g \in G$.
(ii) For all $g \in G$, there exists $g^{-1} \in G$ such that $g \cdot g^{-1} = e = g^{-1} \cdot g$.
(iii) For all $g_1, g_2, g_3 \in G$, $(g_1 \cdot g_2) \cdot g_3 = g_1 \cdot (g_2 \cdot g_3)$.

2. Let $\tau = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ belong to the symmetric group S_3. Recall that $\tau^3 = I = \sigma^2$ and $\sigma\tau = \tau^2\sigma$, where I denotes the identity element of $S_3 = \{I, \tau, \tau^2, \sigma, \tau\sigma, \tau^2\sigma\}$. Write the following product as an element of S_3 in standard form.

$\sigma^{11}\tau^5\sigma^4\tau^{12}\sigma\tau =$

Solution.

$$\begin{aligned} \sigma^{11}\tau^5\sigma^4\tau^{12}\sigma\tau &= \sigma\tau^2 e e \sigma\tau \\ &= \tau\sigma\tau^2\sigma \\ &= \tau\tau\sigma\sigma \\ &= \tau^2. \end{aligned}$$